FAR BEYOND

MAT122

Logarithmic Derivatives

Part I

Stony Brook University

Derivative of $y = log_b\, x$

$$\frac{d}{dx}\log_b x = \frac{1}{x \cdot \ln b}$$

$$\frac{d}{dx}a^x = a^x \cdot \ln a$$

ex. $\left(\log_7 x\right)' = \dfrac{1}{x \cdot \ln 7}$

ex. $\left(\log x\right)' = \dfrac{1}{x \cdot \ln 10}$

Do: $\left(\log_5 x\right)'$

Do: $\left(5^x\right)'$

Special Case:

$$\frac{d}{dx}\ln x = \frac{d}{dx}\log_e x = \frac{1}{x \cdot \ln e} = \frac{1}{x}$$

($\ln e = 1$)

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

Derivative of $y = ln\ x$ with Chain Rule

ex. $\dfrac{d}{dx}\left[\ln(x^2-5)\right]$

$u = x^2 - 5$

$= \dfrac{d}{dx}[\ln u] \cdot \dfrac{du}{dx}$

$= \dfrac{1}{u} \cdot \dfrac{du}{dx}$

$\dfrac{du}{dx} = 2x$

$= \dfrac{1}{x^2-5} \cdot 2x$

$= \boxed{\dfrac{2x}{x^2-5}}$

ex. $\left(\sqrt{\ln x}\right)'$

$= \left((\ln x)^{1/2}\right)'$ $\qquad u = \ln x$

$= \left(u^{1/2}\right)' \cdot u'$ $\qquad u' = \dfrac{1}{x}$

$= \left(\dfrac{1}{2}u^{-1/2}\right) \cdot u'$

$= \left(\dfrac{1}{2}\ln x^{-1/2}\right) \cdot \dfrac{1}{x}$

$= \dfrac{1}{2} \cdot \dfrac{1}{x} \cdot \dfrac{1}{(\ln x)^{1/2}}$

$= \boxed{\dfrac{1}{2x\sqrt{\ln x}}}$

Log and Exponential Derivatives - Do

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}a^x = a^x \ln a$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}\log_b x = \frac{1}{x \cdot \ln b}$$

Do: differentiate $y = 10^x$

Do: differentiate $y = \ln(5x^2 + 1)$

Do: find $\left(e^{5x^2+1}\right)'$

Double Chain Rule

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

ex: Find $f'(x)$ when $f(x) = \ln(5x^2+1)^4$.

$$f' = (\ln u)' \cdot u'$$

$$= \frac{1}{u} \cdot u'$$

$$= \frac{1}{(5x^2+1)^4} \cdot 40x(5x^2+1)^3$$

cancel common factors

write as single fraction

$$= \boxed{\frac{40x}{5x^2+1}}$$

$u = (5x^2+1)^4$ see how u has an inner function

$= v^4$ $\quad v = 5x^2+1$

$v' = 10x$

$u' = (v^4)' \cdot v'$

$= 4v^3 \cdot v'$

$= 4(5x^2+1)^3 \cdot 10x$ simplify

$u' = 40x(5x^2+1)^3$